**Mass and Energy Balances Applied to Electricity Generation**

EMTH171

Case study 2

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**Introduction**

NZ government has proposed to achieve a 97% (which is handled as 100% in this case study) renewable electricity generation system by 2035, without the use of fossil fuels. To satisfy the balance between power demand and supply in this proposed system, it is therefore of significance to extend the renewable supply, including geothermal, wind power and hydroelectricity. This report investigates the new system mathematically with a working simulation, focusing on its expected behavior within one year. Euler’s method is applied to determine some of the essential variables hourly. The viability is checked in the light of the lake level in both North and South Island, where a mass balance should be ensured. Further optimization will also be discussed.

**Model summary**

The proposed system handles electricity supply and demand in North and South Island separately. The power demand in South Island is expected to be

(1)

which is constant through Year 2035, while in North Island, the power demand can be expressed as

(2)

where =5000h, =1000h, and represents the current time from the beginning of the year 2035, in the unit of h(hour).

For the power supply in both islands, the key parameters and equations are listed (**Table 1**). In this model, all the hydro lakes in each island are treated as a single one, and the initial height of lakes are calculated as the middle level.

**Table1. Key parameters and equations in this system in both islands.**

|  |  |  |
| --- | --- | --- |
|  | North Island | South Island |
| Surface area,  () | 620 | 350 |
| Minimum height  (m) | 355.85 | 402 |
| Maximum height  (m) | 357.25 | 410 |
| Height of generator, (m) | 80 | 0 |
| Geothermal,  (MW) | 1525 | 0 |
| Wind,  (MW) | ( varies in two islands) | |
| The inlet flow,  ( |  |  |
| Hydro spillage flow,  ( |  |  |
| Hydroelectricity,  (MW) |  | Or |
| HVDC link,  (MW) |  | |

In this table, represents the wind turbine capacity, with 4130MW spread over two islands. K is a constant with value of 1.55 , L is the length of the spillway weir and is assumed to be 300 m. represents the height of weir with the same value as the maximum height in the table. and are the current height of lake at a given time *t*, with unit of *m*.

Also, there are four essential differential equations in this model that come from the mass balance of the hydro lakes,

(3)

(4)

(5)

(6)

where and () represent the spillage of North Island and South Island hydro lakes at a given time, *t(h)*. The other parameters in the equations have been listed in Table 1.

The value of the spilt water can be determined,

(7)

where is the water density 998, and g is the constant of gravity 9.8().

**Solution method**

The key equations in the model should be rearranged to the forms where Euler’s method can be applied,

(8)

To put it another way, in each step of the loop for Euler’s method, the inputs for the differential equations (3) ~ (6) should be calculated on the basis of the simple equations in Table 1. The step of the loop should be 1 hour.

**Task 1: A working simulation**

The equations are implemented with Euler’s method in Matlab.

The plot of lake levels and spillage against time can be shown below (Figure 1).







**Figure 1. Lake levels and spillage in each island from 0 to 8760 hours.**

From the figures, the minimum lake level in North Island is 356.55m and is higher than the limit (355.85m). The minimum lake level in South Island is 399 m which is lower than its limit (402m).

In this initial simulation, the volume of the spilled water is 0 ( in both islands.

**Task 2: Simple optimization**

The capacity of wind generation in each island ought to be changed to minimize the dollar value of spilt water. All the constraints above should still be met.

a) After listing a large number of possible values of wind capacity, the minimum dollar value for splitting water is: dollars, when wind turbine capacity reaches 4484 MW.

b) New geothermal capacity is likely to cost about $4500 per installed kW of capacity and new wind turbines cost about $3000 per installed kW.

In this model, comparing the required capacity of the existing 580MW of wind and 960MW of geothermal capacity, the extra cost is 7888.5 million dollars.

**Task 3: What-if analysis**

For further optimization, four different modification have been taken into account:

(a) The generating flow in North Island in the first 3 and last 3 months is 0.9 of inlet flow of the year. In winter, the generating flow is slightly increased.

The increase in winter is handled as 1.05 times. The minimum lake level in South Island is higher than its limit (**Figure 2**).





**Figure 2.** Change of lake level in North and South Island hydro lake in the year in Scenario (a).

The total cost in this scenario is dollars.

(b) The wind flow is 50% of normal for July.

This scenario is not viable for the minimum lake level in South Island is lower than the limit (Figure 3).



**Figure 3.** Change of lake level in South Island hydro lake in the year in Scenario (b).

The charge of spilt water is dollars.

(c) The rain fall (the inlet flow) in the South Island is consistently 90% of normal for the year.

Again, the limit of the lake level is not satisfied (Figure 4).



**Figure 4.** Change of lake level in South Island hydro lake in the year in Scenario (c).

(d) In the first 2 months and last 2 months the generating flow of North Island is 0.8 of flow of inlet flow; at same time the capacity factor is half of normal. In other months the capacity factor becomes 1.3 of normal.

This scenario meets the basic constraints(**Figure 5**).





**Figure 5.** Change of lake level in North and South Island hydro lake in the year in Scenario (d).

The value of the spilt water is dollars, which is also an effective optimization.

**Discussion**

A problem based on the balances is always treated as a system of differential equations. Euler’s method is therefore effective in handling the model discussed in this case study.

From the trails in part 3, it can be found that the decrease in wind flow and rain fall can contribute to the shrink of the lake level that may not meet the constraints, but it sometimes reduce the spillage and therefore the value of the spilt water can be saved.

It is viable to reduce the flow in some months to accumulate water, which is helpful to ensure a relatively stable lake level. In this case, there would neither be too much spillage nor a very low lake level in a certain month.

**Appendix Code**

% EMTH171 // CaseStudy2

% Name: Menghao Zhan // Jiyao Zhu

clear

clc

close all

%====================--Constant Values--======================

g = 9.81; % Coefficient of Gravity(m/s^2)

tarray = 1:1:8760; % One year shown by array (hours)

Stand\_d = 1000; % Standard deviation (h)

Mean = 5000; % Mean of distribution with North Island Power demand (h)

density = 998; % (kg/m^3)

K = 1.55; % (m^0.5\*s^-1)

L = 300; % length of weir (m)

%-------------------Assume NI&SI wind capacity--------------------

%===================--Task1--==========================

wind\_geCA\_NI = 4130/2;%(MW)

wind\_geCA\_SI = 4130/2;%(MW)

%===================--Task1--==========================

%===================--Task3(abcd)--==========================

% wind\_geCA\_NI = Wind\_Capacity; % The value get from task2 after optimisation

% wind\_geCA\_NI = Wind\_Capacity; % The value get from task2 after optimisation

%===================--Task3(abcd)--==========================

%===================--Known Values--==========================

Area\_SI = 350e6; % Surface area of South Island(m^2)

Area\_NI = 620e6; % Surface area of North Island(m^2)

Minh\_SI = 402; % Minimum height of South Island(m)

Minh\_NI = 355.85; % Minimum height of North Island(m)

Maxh\_SI = 410; % Maximum height of South Island(m)

Maxh\_NI = 357.25; % Maximum height of North Island(m)

h\_ge\_SI = 0; % Height of generator of South Island(m)

h\_ge\_NI = 80; % Height of generator of North Island(m)

Av\_flow\_SI = 593; % Average flow of South Island(m^3/s)

Av\_flow\_NI = 345; % Average flow of North Island(m^3/s)

Maxge\_SI = 3590; % Maximum generation(MW)

Maxge\_NI = 1870; % Maximum generation(MW)

E\_NI\_ge = 1525; % Energy of geothermal capacity(MW)

%===================--Initial Values--========================

hSI(1) = (Maxh\_SI+Minh\_SI)/2 % Initial height of SI lake (m)

hNI(1) = (Maxh\_NI+Minh\_NI)/2 % Initial height of NI lake (m)

vNI(1) = 0; % Spillage of NI lake (m^3)

vSI(1) = 0; % Spillage of SI lake (m^3)

P\_SI\_demand = 1940; % SI power demand in MW(Eq14,15)

%=====================--For-loop--==========================

for n=2:tarray(end)

t = n;

% 1 ----------NI power demand in MW(Eq14,15)----------

P\_NI\_demand = 4065 + 1.4e6 \* normpdf(t, Mean, Stand\_d);

% 2 ----------Inlet flow rate into NI&SI lakes(Eq5,6)----------

F\_NI\_inlet = 345 + 73\* sin((2\*pi\*(t-3624))/8760);

F\_SI\_inlet = 593 - 183\* sin((2\*pi\*(t-2320))/8760);

%=====================--Task3-c--==========================

% F\_SI\_inlet = 0.9\*(593 - 183\* sin((2\*pi\*(t-2320))/8760));

%=====================--Task3-c--==========================

F\_NI\_ge = F\_NI\_inlet;

%=====================--Task3-a--==========================

% if t < 8760/4

% F\_NI\_ge = 0.9\*F\_NI\_inlet; %Reducing generation flow

% elseif t < 8760\*3/4

% F\_NI\_ge = 1.1\*F\_NI\_inlet;

% else

% F\_NI\_ge = 0.9\*F\_NI\_inlet;

% end

%=====================--Task3-a--==========================

%=====================--Task3-d--==========================

if t < 8760/6 || 8760\*(5/6)<t

% Reducing generation flow to 0.8 accln water

F\_NI\_ge = 0.8\*F\_NI\_inlet;

F\_SI\_ge = 0.8\*F\_SI\_inlet;

CF = CF / 2;

else

% The generation flow dose not change /the wind power capacity changed

F\_NI\_ge = 1\*F\_NI\_inlet;

F\_SI\_ge = 1\*F\_SI\_inlet;

CF = CF \* 1.3;

% F\_SI\_ge = 1.2\*F\_SI\_inlet;

end

%=====================--Task3-d--==========================

%=====================--Task3-b--==========================

% if t<=5088 && t>4344 % Reducing wind power capacity to 50%

% CF = CF/2;

% end

%=====================--Task3-b--==========================

%=====================--Equations--==========================

% 4 --------The hydro-electric generation in NI(Eq7)----------

P\_NI\_hydro = 0.9\*F\_NI\_ge\*density\*g\*(hNI(n-1)-h\_ge\_NI)/(1e6);

% 6 ---- Power of wind-------

P\_NI\_wind = wind\_geCA\_NI \* CF;

P\_SI\_wind = wind\_geCA\_SI \* CF;

%----Electrical power balances for each Island -------

P\_HVDC = P\_NI\_demand - E\_NI\_ge - P\_NI\_wind - P\_NI\_hydro;

% 4-1--------The hydro-electric generation in SI------------

P\_SI\_hydro = P\_SI\_demand + P\_HVDC - P\_SI\_wind;

% 3 ----------The generating flow for NI&SI(Eq8)----------

F\_SI\_ge = (P\_SI\_hydro\*1e6)/((0.9\*density\*g)\*(hSI(n-1)-h\_ge\_SI));

% 10 ---- Spillway flow for each lake(Eq9) -------

if hNI(n-1) > Maxh\_NI

F\_NI\_spill = K\*L\*(hNI(n-1)-Maxh\_NI)^1.5;

Spill\_NI\_dvdt = F\_NI\_spill\*3600;

% Derivative of NI spill water volume (m^3/h)

else

F\_NI\_spill = 0;

Spill\_NI\_dvdt = 0;

end

if hSI(n-1) > Maxh\_SI

F\_SI\_spill = K\*L\*(hSI(n-1)-Maxh\_SI)^1.5;

Spill\_SI\_dvdt = F\_SI\_spill\*3600;

% Derivative of SI spill water volume (m^3/h)

else

F\_SI\_spill = 0;

Spill\_SI\_dvdt = 0;

end

%-----------Main part----------

dhNIt = (F\_NI\_inlet - F\_NI\_ge - F\_NI\_spill) \* 3600/Area\_NI; % (m^3/h)

dhSIt = (F\_SI\_inlet - F\_SI\_ge - F\_SI\_spill) \* 3600/Area\_SI; % (m^3/h)

%-----------Euler's method-----

hNI(n) = hNI(n-1) + dhNIt; %Current lake level of North Island(m)

hSI(n) = hSI(n-1) + dhSIt; %Current lake level of South Island(m)

vNI(n) = vNI(n-1) + (Spill\_NI\_dvdt);%Current lake volume of NI lake (m^3)

vSI(n) = vSI(n-1) + (Spill\_SI\_dvdt);%Current lake volume of SI lake (m^3)

% Calculate money if the minimum criteria condition passed

%=====================--Task2---==========================

%----Find the optimised wind generated capacity of North Island-------

if check == 0;

Total\_cost = cost\_NI + cost\_SI;

if Total\_cost < A\_money

A\_money = Total\_cost;

Wind\_Capacity = wind\_geCA\_NI;

New\_H\_NI = hNI;

New\_H\_SI = hSI;

New\_spillNI = vNI;

New\_spillSI = vSI;

end

end

%=====================--Task2---==========================

end

%=====================--Task2/3---==========================

% Calculation of total cost from North & South Islands.

cost\_NI = 90\*density\*(vNI(tarray(end)))\*g\*(hNI(n)-h\_ge\_NI)/(3600\*1e6);

cost\_SI = 90\*density\*(vSI(tarray(end)))\*g\*(hSI(n)-h\_ge\_SI)/(3600\*1e6);

total = cost\_NI + cost\_SI

%=====================--Task2/3---==========================

%==============================ploting================================

% figure(1)

% plot(tarray,hNI)

% hold on

% plot(tarray,hSI)

% hold off

% xlabel('Time (h)')

% ylabel('Lake Level (m)')

% legend("North Island","South Island")

%==============================ploting================================

figure(1)

plot(tarray , hNI)

xlabel('Time(h)')

ylabel('North lake-level(m)')

figure(2)

plot(tarray , hSI)

xlabel('Time(h)')

ylabel('South lake-level(m)')

figure(3)

plot(tarray , vSI)

xlabel('Time(h)')

ylabel('South spill-volume(m^3)')

figure(4)

plot(tarray , vNI)

xlabel('Time(h)')

ylabel('North spill-volume(m^3)')

%=====================--Task2/b--==========================

E\_NI\_ge = 1605; % When calculating the cost the power from North Island geothermal is original one

Wind\_cost = (Wind\_Capacity-Extra\_wind)\*Cost\_wt\*trans;

Geo\_cost = (E\_NI\_ge-Extra\_geo)\*Cost\_Ngc\*trans;

Cost\_extra\_geo = (Wind\_cost + Geo\_cost)/1e6

%=====================--Task2/b--==========================